1)

$$A+B=\begin{bmatrix}1&2\\2&1\end{bmatrix}+\begin{bmatrix}-3&-2\\4&2\end{bmatrix}=\begin{bmatrix}1-3&2-2\\2+4&1+2\end{bmatrix}=\begin{bmatrix}-2&0\\6&3\end{bmatrix}$$

$$2\cdot A=2\cdot\begin{bmatrix}1&2\\2&1\end{bmatrix}=\begin{bmatrix}2\cdot1&2\cdot2\\2\cdot2&2\cdot1\end{bmatrix}=\begin{bmatrix}2&4\\4&2\end{bmatrix}$$

$$2A-B=2\cdot\begin{bmatrix}1&2\\2&1\end{bmatrix}-\begin{bmatrix}-3&-2\\4&2\end{bmatrix}=\begin{bmatrix}2\cdot1+3&2\cdot2+2\\2\cdot2-4&2\cdot1-2\end{bmatrix}$$

$$=\begin{bmatrix}5&6\\0&0\end{bmatrix}$$

3)

$$2\cdot A-5B=2\cdot\begin{bmatrix}4&11&-9\\0&3&2\\-3&1&1\end{bmatrix}-5\cdot\begin{bmatrix}1&2&-7\\-4&6&11\\-6&4&9\end{bmatrix}$$

$$=\begin{bmatrix}2\cdot4-5\cdot1&2\cdot11-5\cdot2&2\cdot(-9)-5\cdot(-7)\\2\cdot0-5\cdot(-4)&2\cdot3-5\cdot6&2\cdot2-5\cdot11\\2\cdot(-3)-5\cdot(-6)&2\cdot1-5\cdot4&2\cdot1-5\cdot9\end{bmatrix}$$

$$=\begin{bmatrix}3&12&17\\20&-24&-51\\24&-18&-43\end{bmatrix}$$

$-51 \to c_{23}$

$-18 \to c_{32}$

4)

$$\begin{bmatrix}4a&4b\\4c&-4\end{bmatrix}=\begin{bmatrix}2b&2c\\-2a&2\end{bmatrix}+\begin{bmatrix}8&2a\\10&-2a\end{bmatrix}$$

$$\begin{bmatrix}4a&4b\\4c&-4\end{bmatrix}=\begin{bmatrix}2b+8&2c+2a\\10-2a&2-2a\end{bmatrix}$$

$4c=10-2a \Rightarrow 4c=10-2\cdot3 \Rightarrow \boxed{c=1}$

$-4=2-2a \Rightarrow \boxed{a=3}$

$4a=2b+8 \Rightarrow$

$4b=2c+2a \Rightarrow 4b=2\cdot1+2\cdot3=8$

$\boxed{b=2}$

7)

d)

$$\underbrace{\begin{bmatrix}2&-1\\1&4\end{bmatrix}}_{A}\cdot\underbrace{\begin{bmatrix}0&0\\3&-3\end{bmatrix}}_{B}=\begin{bmatrix}2\cdot0+(-1)\cdot3&2\cdot0+1\cdot3\\1\cdot0+4\cdot3&1\cdot0+4\cdot(-3)\end{bmatrix}$$

$$=\begin{bmatrix}-3&3\\12&-12\end{bmatrix}$$

5_{21}

b)

$$B\cdot A=\begin{bmatrix}0&0\\3&-3\end{bmatrix}\cdot\begin{bmatrix}2&-1\\1&4\end{bmatrix}=\begin{bmatrix}0\cdot2+0\cdot1&0\cdot(-1)+0\cdot4\\3\cdot2-3\cdot1&3\cdot(-1)-3\cdot4\end{bmatrix}$$

$$=\begin{bmatrix}0&0\\3&-15\end{bmatrix}$$

$$(B\cdot A)^T=\begin{bmatrix}0&0\\3&-5\end{bmatrix}^T=\begin{bmatrix}0&3\\0&-5\end{bmatrix}$$

b)

$$\begin{bmatrix}1&-1&7\\2&-1&8\\3&1&-1\end{bmatrix}\cdot\begin{bmatrix}1&1&2\\2&1&1\\1&-3&2\end{bmatrix}=$$

$$=\begin{bmatrix}1\cdot1-1\cdot2+7\cdot1&1\cdot1-1\cdot1+7\cdot(-3)&1\cdot2-1\cdot1+7\cdot2\\2\cdot1-1\cdot2+8\cdot1&2\cdot1-1\cdot1+8\cdot(-3)&2\cdot2-1\cdot1+8\cdot2\\3\cdot1+1\cdot2-1\cdot1&3\cdot1+1\cdot1+1\cdot3&3\cdot2+1\cdot1-1\cdot2\end{bmatrix}$$

$$=\begin{bmatrix}6&-21&15\\8&-23&19\\4&7&5\end{bmatrix}$$

13) a)

$$\begin{bmatrix}1&2\\3&7\end{bmatrix}^{-1}=?$$

$$A^{-1}=\frac{1}{|A|}\cdot\begin{bmatrix}7&-2\\-3&1\end{bmatrix}=\frac{1}{\underbrace{1\cdot7-3\cdot2}_{1}}\cdot\begin{bmatrix}7&-2\\-3&1\end{bmatrix}$$

12)

b)

$$A=\begin{bmatrix}2&-17&11\\-1&11&-7\\0&3&-2\end{bmatrix}$$

$a_{11}=\begin{vmatrix}11&-7\\3&-2\end{vmatrix}=-22+21=-1$

$a_{12}=\begin{vmatrix}-1&-7\\0&-2\end{vmatrix}=2+0=2$

$a_{13}=\begin{vmatrix}-1&11\\0&3\end{vmatrix}=-3-0=-3$

$a_{21}=\begin{vmatrix}-17&11\\3&-2\end{vmatrix}=34-33=1$ $\quad a_{31}=\begin{vmatrix}-17&11\\11&-7\end{vmatrix}=119-121=-2$

$a_{22}=\begin{vmatrix}2&11\\0&-2\end{vmatrix}=-4-0=-4$ $\quad a_{32}=\begin{vmatrix}2&11\\-1&-7\end{vmatrix}=-14+11=-3$

$a_{23}=\begin{vmatrix}2&-17\\0&3\end{vmatrix}=6-0=6$ $\quad a_{33}=\begin{vmatrix}2&-17\\-1&11\end{vmatrix}=22-17=5$

$$adj(A)=\begin{bmatrix}a_{11}&-a_{12}&a_{13}\\-a_{21}&a_{22}&-a_{23}\\a_{31}&-a_{32}&a_{33}\end{bmatrix}^T=\begin{bmatrix}-1&-2&-3\\-1&-4&-6\\-2&3&5\end{bmatrix}^T=\begin{bmatrix}-1&-1&-2\\-2&-4&3\\-3&-6&5\end{bmatrix}$$

$$Det(A)=|A|=2\cdot(-22+21)+17\cdot(2-0)+11\cdot(-3+0)=-1$$

$$A^{-1}=\frac{1}{|A|}\cdot adj(A)=\frac{1}{-1}\cdot\begin{bmatrix}-1&-1&-2\\-2&-4&3\\-3&-6&5\end{bmatrix}=\begin{bmatrix}1&1&2\\2&4&-3\\3&6&-5\end{bmatrix}$$

8) c) $A=\begin{bmatrix}-1 & 3\\ 4 & -5\\ 0 & 2\end{bmatrix}$, $B=\begin{bmatrix}1 & 2\\ 0 & 3\end{bmatrix}$

$(A\cdot B)^T = ?$, $(B\cdot A)^{-1} = ?$

$$A\cdot B=\begin{bmatrix}-1 & 3\\ 4 & -5\\ 0 & 2\end{bmatrix}\cdot\begin{bmatrix}1 & 2\\ 0 & 3\end{bmatrix}=\begin{bmatrix}-1\cdot1+3\cdot0 & -1\cdot2+3\cdot3\\ 4\cdot1-5\cdot0 & 4\cdot2-5\cdot3\\ 0\cdot1+2\cdot0 & 0\cdot2+2\cdot3\end{bmatrix}$$

$$A\cdot B=\begin{bmatrix}-1 & 7\\ 4 & -7\\ 0 & 6\end{bmatrix}$$

$$(A\cdot B)^T=\begin{bmatrix}-1 & 7\\ 4 & -7\\ 0 & 6\end{bmatrix}^T=\begin{bmatrix}-1 & 4 & 0\\ 7 & -7 & 6\end{bmatrix}$$

$$B\cdot A=\begin{bmatrix}1 & 2\\ 0 & 3\end{bmatrix}\cdot\begin{bmatrix}-1 & 3\\ 4 & -5\\ 0 & 2\end{bmatrix}$$

2×2 $\quad 3\times 2$

$2\neq 3$ cannot be multiplied.

17) similar

$2x+3y+z=2$

$-x+2y+3z=-1$

$-3x-3y+z=0$

$$\det\begin{bmatrix}2 & 3 & 1\\ -1 & 2 & 3\\ -3 & -3 & 1\end{bmatrix}=2\cdot(2+9)-3\cdot(-1+9)+1\cdot(3+6)=22-24+9=7$$

$$x=\frac{\begin{vmatrix}2 & 3 & 1\\ -1 & 2 & 3\\ 0 & -3 & 1\end{vmatrix}}{7}=\frac{2\cdot(2+9)-3\cdot(-1)+1\cdot3}{7}=\frac{28}{7}=4$$

$$y=\frac{\begin{vmatrix}2 & 2 & 1\\ -1 & -1 & 3\\ -3 & 0 & 1\end{vmatrix}}{7}=\frac{2\cdot(-1)-2\cdot(-1+9)+1\cdot(-3)}{7}=-\frac{21}{7}=-3$$

$$z=\frac{\begin{vmatrix}2 & 3 & 2\\ -1 & 2 & -1\\ -3 & -3 & 0\end{vmatrix}}{7}=\frac{2\cdot(-3)-3\cdot(-3)+2\cdot(3+6)}{7}=\frac{21}{7}=3$$

7b) $x+y+z=0$

$2x+z-y=-1$

$-z+3y-x=-8$

$x+y+z=0$

$2x-y+z=-1$

$-x+3y-z=-8$

$A=\begin{bmatrix}1 & 1 & 1\\ 2 & -1 & 1\\ -1 & 3 & -1\end{bmatrix}$, $|A|=1\cdot(1-3)-1\cdot(-2+1)+1\cdot(6-1)$

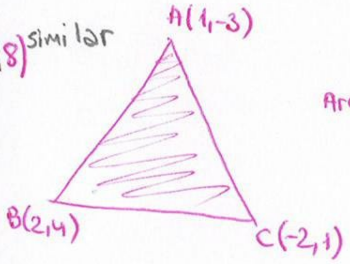
$|A|=-2+1+5=4$

$$x=\frac{\begin{vmatrix}0 & 1 & 1\\ -1 & -1 & 1\\ 8 & 3 & -1\end{vmatrix}}{5}=\frac{0-1\cdot(1-8)+1\cdot(-3+8)}{4}=\frac{12}{4}=3$$

$$y=\frac{\begin{vmatrix}1 & 0 & 1\\ 2 & -1 & 1\\ -1 & 8 & -1\end{vmatrix}}{5}=\frac{1\cdot(1-8)-0+1\cdot(16-1)}{4}=\frac{8}{4}=2$$

$$z=\frac{\begin{vmatrix}1 & 1 & 0\\ 2 & -1 & -1\\ -1 & 3 & 8\end{vmatrix}}{5}=\frac{1\cdot(8+3)-1\cdot(16-1)+0}{4}=-\frac{20}{4}=-5$$

18) similar



$$\text{Area}=\mp\frac{1}{2}\cdot\begin{vmatrix}1 & -3 & 1\\ 2 & 4 & 1\\ -2 & 1 & 1\end{vmatrix}$$

$$\text{Area}=\frac{1}{2}\cdot\left[1(4-1)+3\cdot(2+2)+1\cdot(2+8)\right]$$

$$=\frac{1}{2}\cdot(3+12+10)=\frac{25}{2}\text{ unit}^2$$

20) similar equ. of line passing through

d) $A(3,1)$, $B(2,-4)$

$$\begin{vmatrix}x & y & 1\\ 3 & 1 & 1\\ 2 & -4 & 1\end{vmatrix}=x\cdot(1+4)-y\cdot(3-2)+1\cdot(-12-2)$$

$$\boxed{5x-y-14=0}$$

19) similar Show whether collinear or not

$(2,6)$, $(-5,10)$, $(15,-2)$

if $|A|=\begin{vmatrix}2 & 6 & 1\\ -5 & 10 & 1\\ 15 & -2 & 1\end{vmatrix}=0$ collinear, if $|A|\neq0$ not collinear.

$|A|=2\cdot(10+2)-6\cdot(-5-15)+1\cdot(10-150)$

$=24-180-140=24-320=-296$

not collinear.

Q5

1) Find A+B, 2A and 2A−B

if $A=\begin{bmatrix}1&2\\2&1\end{bmatrix}$, $B=\begin{bmatrix}-3&-2\\4&2\end{bmatrix}$

2) Find C_{23} and C_{32} where

$C = 2A-5B$ if

$A=\begin{bmatrix}4&11&-9\\0&3&2\\-3&1&1\end{bmatrix}$ and $B=\begin{bmatrix}1&2&-7\\-4&6&11\\-6&4&9\end{bmatrix}$

3) Solve for a, b and c

$$4\begin{bmatrix}a&b\\c&-1\end{bmatrix}=2\cdot\begin{bmatrix}b&c\\-a&1\end{bmatrix}+2\cdot\begin{bmatrix}4&a\\5&-a\end{bmatrix}$$

4)
a) $A=\begin{bmatrix}2&-1\\1&4\end{bmatrix}$, $B=\begin{bmatrix}0&0\\3&-3\end{bmatrix}$

$A\cdot B=?$, $(B\cdot A)^T=?$

b) $A=\begin{bmatrix}1&-1&7\\2&-1&8\\3&1&-1\end{bmatrix}$, $B=\begin{bmatrix}1&1&2\\2&1&1\\1&-3&2\end{bmatrix}$

$A\cdot B=?$

5) Find inverse of each

a) $\begin{bmatrix}1&2\\3&7\end{bmatrix}$, $\begin{bmatrix}1&-2\\2&-3\end{bmatrix}$

b) $\begin{bmatrix}2&-17&11\\-1&11&-7\\0&3&-2\end{bmatrix}$

6) $A=\begin{bmatrix}-1&3\\4&-5\\0&2\end{bmatrix}$ $B=\begin{bmatrix}1&2\\0&7\end{bmatrix}$

Find $(A\cdot B)^T$ and $(B\cdot A)^{-1}$.

7) Solve the system of linear equations using cramer's Rule

$2x+3y+z=2$
$-x+2y+3z=-1$
$-3x-3y+z=0$,

$x+y+z=0$
$2x+z-y=-1$
$-z+3y-x=-8$

8) Find the area of the triangle surrounded by the lines which passes through the points A(1,−3), B=(2,4) and C(−2,1)

9) a) Find the equation of the line passing through the points A(3,1) and B(2,−4)

b) show ~~that~~ whetler the points are collinear or not

(2,6), (−5,10), (15,−2)
(2,10), (6,13), (−6,4)
(−2,5), (7,2), (3,−4)

10) Find the determinant of the matrices

$\begin{bmatrix}1&0&4\\-2&1&0\\3&2&1\end{bmatrix}$, $\begin{bmatrix}2&1&-1\\0&4&3\\5&0&-2\end{bmatrix}$, $\begin{bmatrix}3&-1\\2&4\end{bmatrix}$....

5 5

sınav örneği